JEE(Main) 2025 | DATE: 28-01-2025 (SHIFT-1) | PAPER-1 | MEMORY BASED

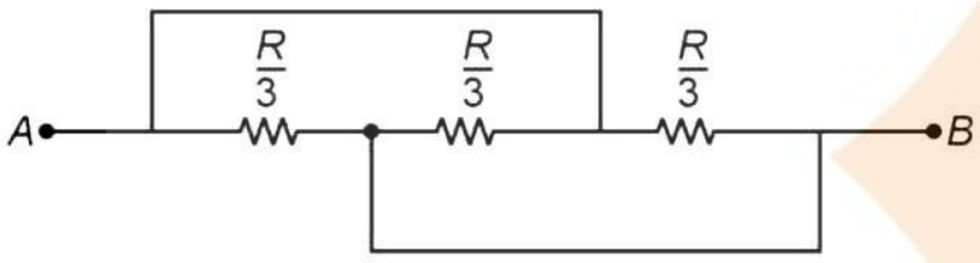
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Find the equivalent resistance across *A* & *B*.



(1) R (2) $R/6$

(3) $R/3$ (4) $R/9$

Answer (4)

Sol. Equivalent circuit can be redrawn as

$$R_{eq} = \frac{1}{3}\left(R/3\right) = \frac{R}{9}$$

2. A uniform wire of linear charge density λ is placed along *y*-axis. The locus of equipotential surface is

(1) $x^2 + y^2 + z^2$ = constant

(2) $x^2 + z^2$ = constant

(3) xyz = constant

(4) $xy + yz + zx$ = constant

Answer (2)

Sol. Concentric cylinders are the equipotential surface.

3. Which of following reaction is correct?

(Where symbols have their usual meanings)

(1) $n \rightarrow p + e^- + \nu$ (2) $n \rightarrow p + e^+ + \nu$

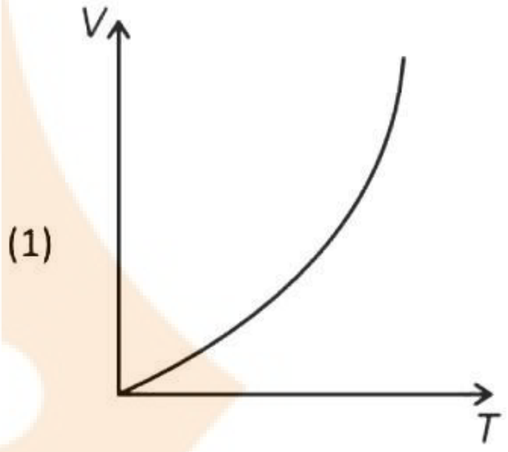
(3) $n \rightarrow p + e^+ + \bar{\nu}$ (4) $n \rightarrow p + e^- + \bar{\nu}$

Answer (4)

Sol. In 2, 3 charge conservation is not holding and in neutron decay, antineutrino $(\bar{\nu})$ is released.

4. The graph of root mean square velocity *v/s* temperature is

(1)



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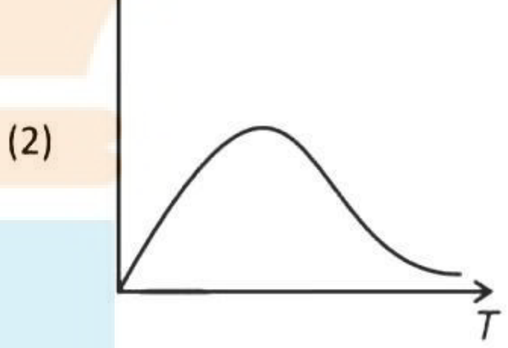

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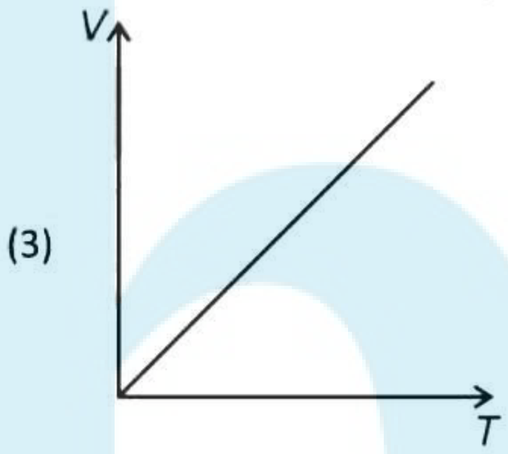

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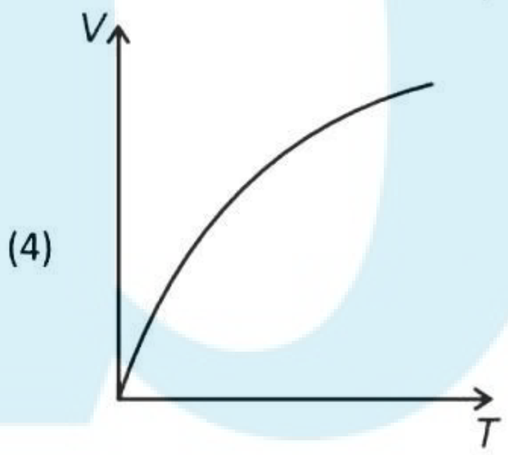
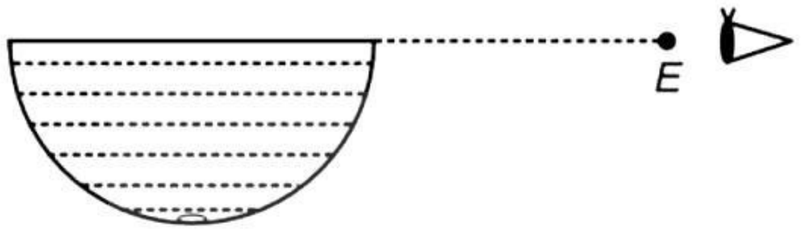


Answer (4)

Sol. We know $\frac{1}{2}MV_{Rms}^2 = \frac{3}{2}RT$

$$\Rightarrow V_{RMS} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{RMS} \propto \sqrt{T}$$

5. A coin is placed at the bottom of a hemispherical container filled with a liquid of refractive index μ. Find the least refractive index if the coin is visible to an observer at *E*.

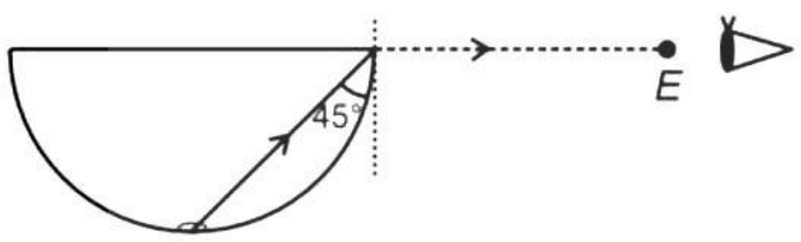


(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $3\sqrt{2}$

Answer (2)

Sol.

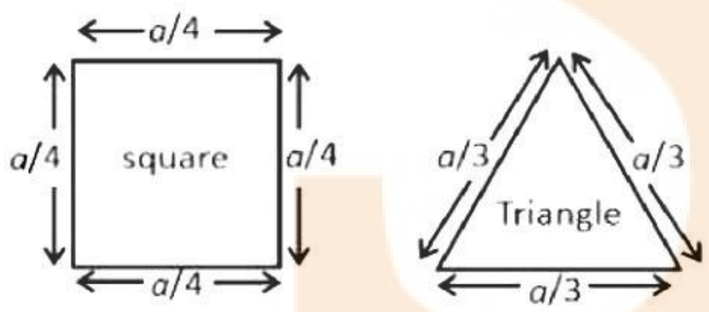


$\theta_c = 45°$

$$\sin 45° = \frac{1}{\mu}$$

$$\mu = \frac{1}{\sin 45°} = \sqrt{2}$$

6. In the given figure, the square and the triangle have same resistance per unit length. Find the ratio of their resistances about adjacent corners.



(1) $\frac{32}{27}$ (2) $\frac{27}{32}$

(3) $\frac{8}{9}$ (4) $\frac{9}{8}$

Answer (2)

Sol. Let the resistance per unit length be λ, then

For square,

$$R_{Square} = \frac{\left(\frac{3\lambda a}{4}\right)\left(\frac{\lambda a}{4}\right)}{\frac{3\lambda a}{4} + \frac{\lambda a}{4}} = \frac{3\lambda a}{16}$$

For triangle,

$$R_{Triangle} = \frac{\left(\frac{2\lambda a}{3}\right)\left(\frac{\lambda a}{3}\right)}{\frac{2\lambda a}{3} + \frac{\lambda a}{3}} = \frac{2\lambda a}{9}$$

$$\frac{R_{Square}}{R_{Triangle}} = \frac{3\lambda a}{16} \times \frac{a}{2\lambda a} = \frac{27}{32}$$

7. **Assertion :** Work done by central force is independent of path.

Reason : Potential energy is associated with every force.

(1) Both Assertion and Reason are correct

(2) Assertion is correct, Reason is incorrect

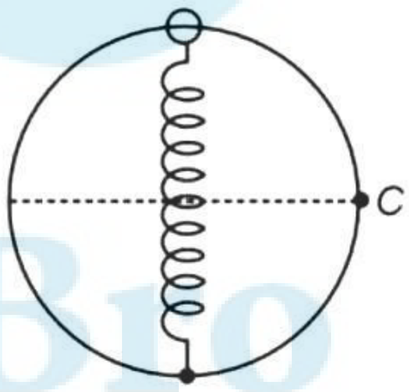
(3) Assertion is incorrect, Reason is correct

(4) Both Assertion and Reason are incorrect

Answer (4)

Sol. Not all central force/s are conservative so work done by central force might depend on path.

8. There is smooth ring of radius *R* in vertical plane. A spring of natural length *R* & elastic constant *K* is vertical along a diameter. The free end is connected to bead of mass *m* & when slightly disturbed it reaches point *C* with speed v where v is



(1) $\sqrt{\frac{KR^2\left(\sqrt{2}-1\right)+2mgR}{m}}$

(2) $\sqrt{\frac{2KR^2\left(\sqrt{2}-1\right)+2mgR}{m}}$

(3) $\sqrt{\frac{2KR^2\left(\sqrt{2}-1\right)+mgR}{m}}$

(4) $\sqrt{\frac{KR^2\left(\sqrt{2}-1\right)+mgR}{m}}$

Answer (2)

Sol. Loss in PE = gain in KE

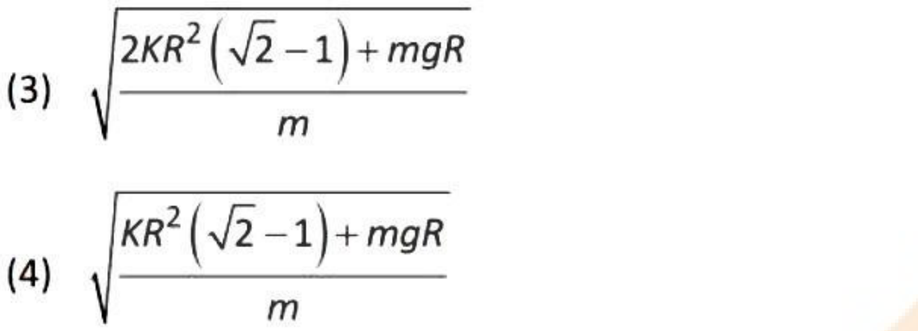
$$\frac{1}{2}K\left(R^2\right)-\frac{1}{2}K\left(\sqrt{2}-1\right)^2R^2+mgR=\frac{1}{2}mv^2$$

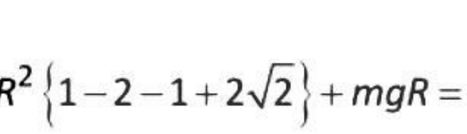
$$\frac{1}{2}KR^2\left\{1-2-1+2\sqrt{2}\right\}+mgR=\frac{1}{2}mv^2$$

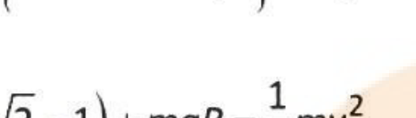
$$KR^2\left(\sqrt{2}-1\right)+mgR=\frac{1}{2}mv^2$$

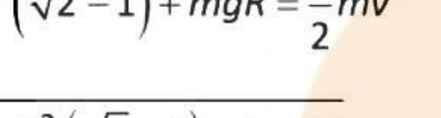
$$\sqrt{\frac{2KR^2\left(\sqrt{2}-1\right)+2mgR}{m}}=v$$

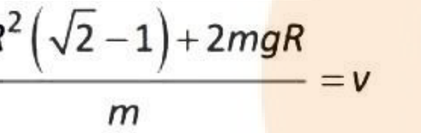
9. The equivalent logic gate for the circuit shown below is



(1) 

(2) 

(3) 

(4) 

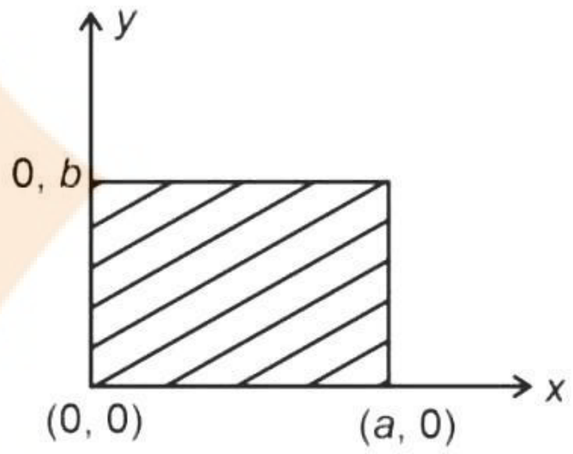
Answer (2)

Sol. $Y=\overline{A\bar{B}+AB}$

$$Y=\overline{A\left(\bar{B}+B\right)}$$

$$Y=\bar{A}$$

10. Surace mass density varies as $\sigma=\frac{\sigma_0 x}{ab}$ for the given plane sheet. Find the position of centre of mass for the distribution given



(1) $\frac{2a}{3},\frac{2b}{3}$

(2) $\frac{2a}{3},\frac{b}{2}$

(3) $\frac{a}{3},\frac{b}{2}$

(4) $\frac{a}{2},\frac{b}{2}$

Answer (2)

Sol. As there is no variation of mass density in y direction. So

$$y_{cm}=\frac{b}{2}.$$

Now for x direction

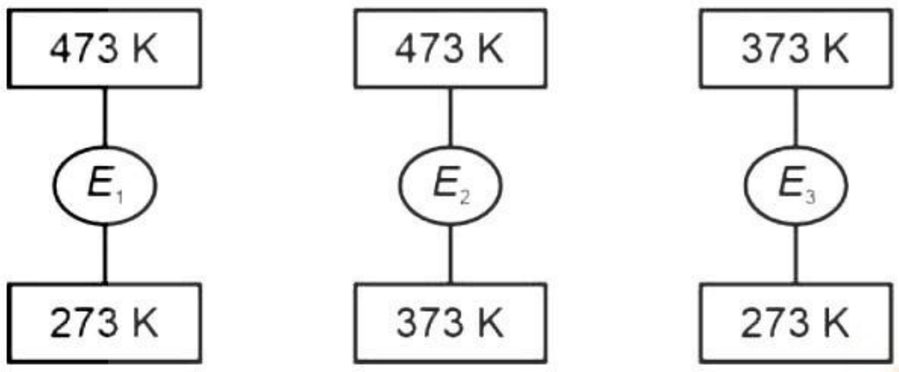
$$dm=\left(\frac{\sigma_0 x}{ab}\right)b.dx$$

$$x_{cm}=\frac{\int_0^a xdm}{\int_0^a dm}=\frac{\frac{\sigma_0}{a}\int_0^a x^2dx}{\frac{\sigma_0}{a}\int xdx}$$

$$\Rightarrow\quad x_{cm}=\frac{a^3\times 2}{3\times a^2}=\frac{2}{3}a$$

So $r_{cm}=\frac{2}{3}a,\frac{b}{2}$

11. η_1, η_2 and η_3 are the efficiencies of the three Carnot engines E_1, E_2 and E_3 operating between temperatures shown in the figure. Choose the correct option relating the efficiencies.



(1) $\eta_2 + \eta_3 > \eta_1$ (2) $\eta_2 + \eta_3 = \eta_1$

(3) $\eta_2 + \eta_3 < \eta_1$ (4) $\eta_1 + \eta_2 = \eta_3$

Answer (1)

Sol. $\eta_1 = 1 - \frac{273}{473} = \frac{200}{473}$

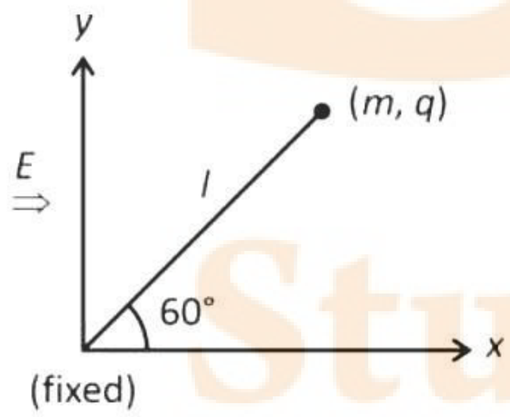
$\eta_2 = 1 - \frac{373}{473} = \frac{100}{473}$

$\eta_3 = 1 - \frac{273}{373} = \frac{100}{373}$

$\eta_1 - \eta_2 = \frac{100}{473} < \eta_3$

i.e., $\eta_2 + \eta_3 > \eta_1$

12. A simple pendulum of length l and bob of mass m is placed on smooth horizontal surface as shown. When electric field of strength E is switched on, the bob passes the x-axis with speed v then



(1) $v = \sqrt{\frac{2qEl}{m}}$ (2) $v = \sqrt{\frac{qEl}{m}}$

(3) $v = \sqrt{\frac{qEl}{2m}}$ (4) $v = 2\sqrt{\frac{qEl}{m}}$

Answer (2)

Sol. $W = \Delta K$

$qE(l - l\cos 60^\circ) = \frac{1}{2}mv^2$

$\frac{qEl}{m} = v^2$

13. **Statement-I :** Velocity of sound in solids is more compared to that in gases.

Statement-II : Bulk modules of gas is more than that of solids.

(1) Statement-I is correct statement-II is correct

(2) Statement-I is correct statement-II is incorrect

(3) Statement-I is incorrect statement-II is correct

(4) Statement-I is incorrect statement-II is incorrect

Answer (2)

Sol. Speed of sound in medium depends on elastic and inertia properly of medium.

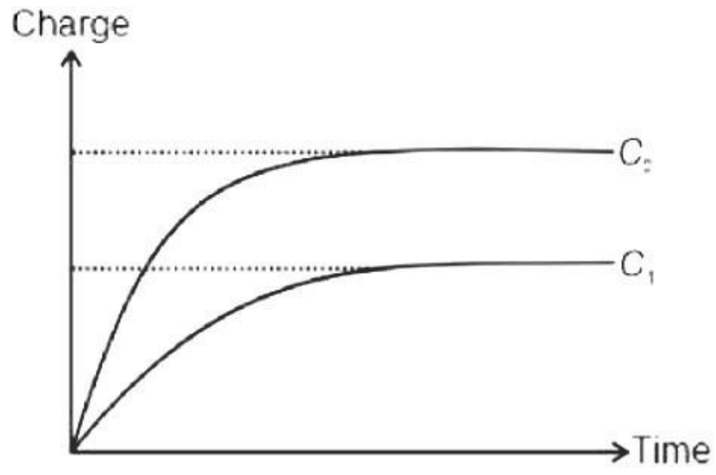
for gas $v = \sqrt{B/\rho}$

for solids $v = \sqrt{Y/\rho}$

The elastic properly of solids happens to be many fold greater than that of elastic properly of gases.

Bulk modulus of gas depends on the process, $B = \frac{-\Delta P}{\Delta V / V}$, which varies between 0 to ∞ therefore in general statement 2 is incorrect.

14. To capacitor C_1 and C_2 are connected across same battery and store energies U_1 and U_2 respectively at steady state. Choose the correct option by observing the graph of charge vs time shown below.



(1) $C_1 > C_2$, $U_1 > U_2$

(2) $C_1 < C_2$, $U_1 < U_2$

(3) $C_1 > C_2$, $U_1 < U_2$

(4) $C_1 < C_2$, $U_1 > U_2$

Answer (2)

Sol. Steady state

$$\frac{C_1}{C_2} = \frac{Q_1}{Q_2} < 1$$ (Connected across same potential difference)

$$\frac{C_1}{C_2} = \frac{U_1}{U_2} < 1$$

15. Energy of photon of wavelength λ is E_0 which is equal to kinetic energy of proton of mass m_p. The ratio of de Broglie wavelengths of proton and photon is

(1) $\frac{1}{c}\sqrt{\frac{2E_0}{m_p}}$

(2) $\frac{1}{c}\sqrt{\frac{E_0}{2m_p}}$

(3) $\frac{2}{c}\sqrt{\frac{E_0}{m_p}}$

(4) $\frac{1}{2c}\sqrt{\frac{E_0}{m_p}}$

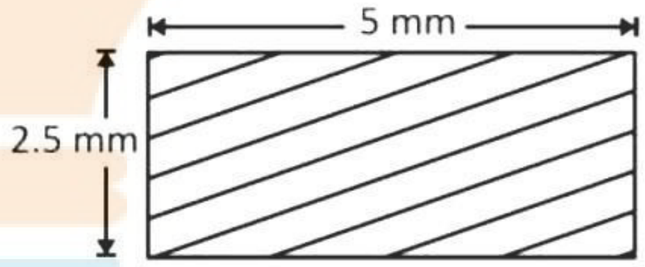
Answer (2)

Sol. $\frac{1}{2}m_p v^2 = E_0 \Rightarrow p = \sqrt{2m_p E_0}$ $\quad \lambda_p = \frac{h}{\sqrt{2m_p E_0}}$

for photon $\frac{hc}{\lambda} = E_0 \Rightarrow \lambda_{ph} = \frac{hc}{E_0}$

$$\Rightarrow \frac{\lambda_p}{\lambda_{ph}} = \frac{h}{\sqrt{2m_p E_0}} \times \frac{E_0}{hc} = \frac{1}{c}\sqrt{\frac{E_0}{2m_p}}$$

16. The lengths of a rectangular sheet is measured from a screw gauge of pitch 0.75 mm and number of division on circular scale = 15. Find maximum possible error in measurement of area.



(1) 0.225 mm^2

(2) 0.375 mm^2

(3) 0.75 mm^2

(4) 0.30 mm^2

Answer (2)

Sol. Lest count of screw gauge

$$= \frac{0.75}{15} = 0.05 \text{ mm}$$

Now, $S = lb$

$$\Rightarrow \frac{\Delta S}{S} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\Rightarrow \frac{\Delta S}{S} = \left(\frac{0.05}{5}\right) + \frac{0.05}{2.5} = \frac{1}{100} + \frac{2}{100} = \frac{3}{100}$$

$$\Delta S = \left(\frac{3}{100}\right) \times 5 \times 2.5 = 0.375 \text{ mm}^2$$

17. There are two prisms of refractive indices of 1.54 & 1.72 respectively. If ray is not deviating after passing through two prisms, then find prism angle of second prism if prism angle of first prism is 4°

(1) 2° (2) 3°

(3) 4° (4) 3.5°

Answer (2)

Sol. $S_1 = S_2$

$\Rightarrow (1.54 - 1)4° = (1.72 - 1)\,A.$

$\Rightarrow A = 3°$

18. The energy associated with a cylindrical region due to an EM wave $E = 100\sin(kx - \omega t)$ is U_0. Find the equation of EM wave for which a cylinder of same length and half the diameter (as previous one) contains same energy U_0.

(1) $200\sin(kx - \omega t)$ (2) $25\sin(\omega t - kx)$

(3) $50\sin(kx - \omega t)$ (4) $400\sin(\omega t - kx)$

Answer (1)

Sol. For EM wave $E = 100\sin(\omega t - kx)$

Energy density $\rho_{a_v} = \frac{1}{2}\epsilon_0 (E_0)^2$

Initial volume $v_0 = \pi R^2 l$

Volume in next situation = $v = \frac{\pi R^2 l}{4} = \frac{V_0}{4}$

So, $u_0 = \frac{1}{2}\epsilon_0 (E_0)^2 \times V_0 = \frac{1}{2}\epsilon_0 (E_1^2)\cdot\frac{V_0}{4}$

$\Rightarrow E_0^2 = \frac{E_1^2}{4} \Rightarrow E_1 = 2E_0 = 200\ \text{V/m}$

So, required equation will have 200 V/m of amplitude.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The dimensions of Young's modules of elasticity per unit length is $M^aL^bT^c$ then $|a + b + c|$ is

Answer (3)

Sol. $\frac{Y}{L} = \frac{MLT^{-2}}{L^2}\frac{1}{L} = ML^{-2}T^{-2}$

$|1 - 2 - 2| = 3$

22. In a *YDSE*, the distance of the 10th bright fringe from the central maxima is 10 mm when light of wavelength used is 600 nm. Find the distance (in mm) of the 10th bright fringe from the central maxima if light of wavelength 660 nm is used instead.

Answer (11)

Sol. $y = 10\frac{\lambda D}{d}$

$\frac{y_2}{y_1} = \frac{\lambda_2}{\lambda_1}$

$y_2 = \frac{\lambda_2}{\lambda_1} \times y_1$

$= \frac{660\ \text{nm}}{600\ \text{nm}} \times 10\ \text{mm}$

$= 11\ \text{mm}$

23.

24.

25.

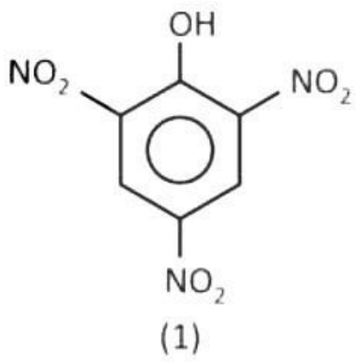
CHEMISTRY

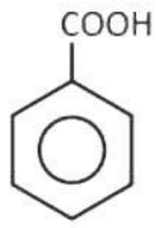
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. What is the rate of reaction for releasing $CO_2(g)$ with aq. $NaHCO_3$ among following?


(1)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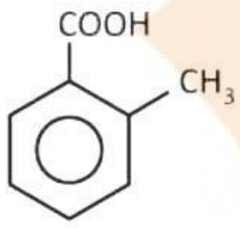

(3)

(1) (1) > (2) > (3) (2) (3) > (2) > (1)

(3) (1) > (3) > (2) (4) (2) > (3) > (1)

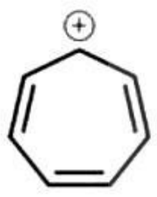
Answer (3)

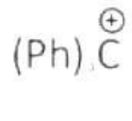
Sol. pK_a (Benzoic acid) : 4.27

pK_a (o-Toluic acid) : 3.91 → due to ortho effect.

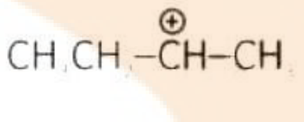
pK_a (Picric acid) : 0.3

2. Consider the following carbocations


(i)


(ii)


(iii)


(iv)

The correct increasing order of stability of these carbocations is:

(1) (i) < (ii) < (iii) < (iv) (2) (iv) < (iii) < (ii) < (i)

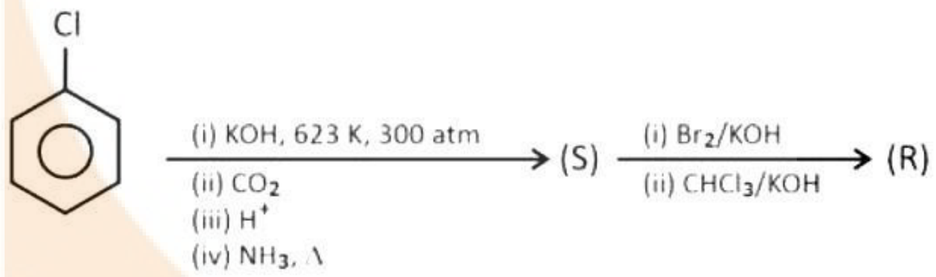
(3) (ii) < (iii) < (iv) < (i) (4) (iv) < (iii) < (i) < (ii)

Answer (2)

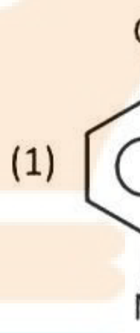
Sol. Tropylium carbocation is most stable due to presence of aromaticity.

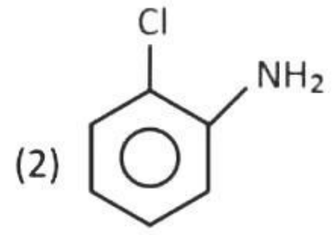
∴ (i) > (ii) > (iii) > (iv) is the correct order of stability of carbocation.

3. In the given reaction sequence:



The compound R is


(1)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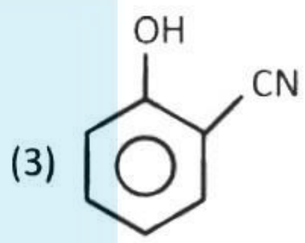
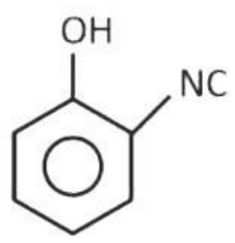
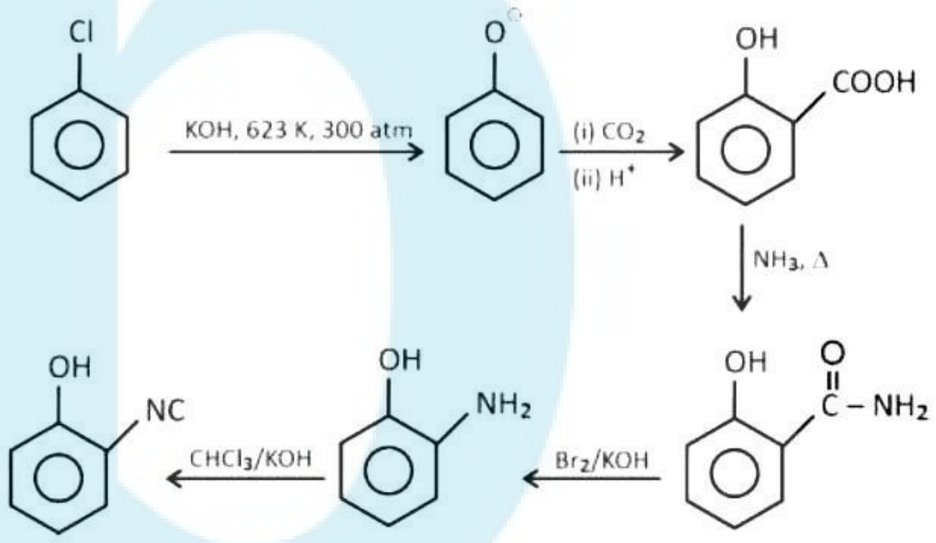

(3)


(4)

Answer (4)

Sol.



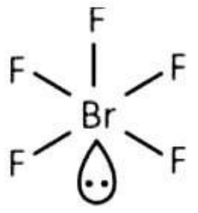
4. Which of the following pair have square pyramidal shape?

(1) BrF_5, $XeOF_4$ (2) SbF_5, BrF_5

(3) PCl_5, $XeOF_4$ (4) PCl_5, SbF_5

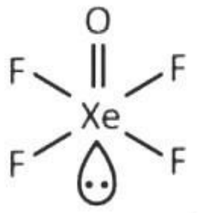
Answer (1)

Sol. $BrF_5 \Rightarrow sp^3d^2$



Square pyramidal

$XeOF_4 \Rightarrow sp^3d^2$



Square pyramidal

5. Which of the following set of quantum numbers have same energy?

(a) n = 2, l = 2, m = +1

(b) n = 2, l = 1, m = –1

(c) n = 3, l = 2, m = 0

(d) n = 3, l = 2, m = 1

(1) a, b

(2) b, c

(3) c, d

(4) a, c

Answer (3)

Sol.: (a) $\Rightarrow$ 2d = does not exist

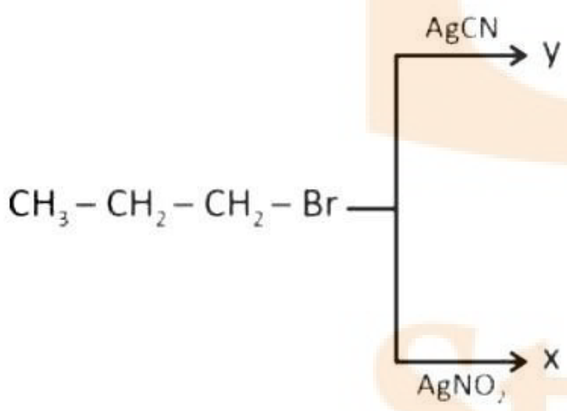
(b) $\Rightarrow$ 2p

(c) $\Rightarrow$ 3d same energy

(d) $\Rightarrow$ 3d same energy

The value of n + l is same for (c) and (d) both represents 3d orbital

6. Consider the following reaction



The major product x and y respectively are

(1) $CH_3CH_2CH_2ONO$ & $CH_3CH_2CH_2CN$

(2) $CH_3CH_2CH_2NO_2$ & $CH_3CH_2CH_2CN$

(3) $CH_3CH_2CH_2NO_2$ & $CH_3CH_2CH_2NC$

(4) $CH_3CH_2CH_2ONO$ & $CH_3CH_2CH_2CN$

Answer (3)

Sol. CN^- and NO_2^- are ambidentate nucleophile but since AgCN and $AgNO_2$ are covalent compound, so only the nitrogen can donate electrons.

7. Match the following column and choose the correct option.

	Column-I		**Column-II**
(A)	$H_2O_2 \rightarrow H_2O + O_2$	(P)	Combustion reaction
(B)	$NaH \rightarrow Na + H_2$	(Q)	Disproportionation reaction
(C)	$CH_4 + O_2 \rightarrow CO_2 + H_2O$	(R)	Decomposition reaction
(D)	$Fe + CuSO_4 \rightarrow FeSO_4 + Cu$	(S)	Displacement reaction

(1) A–(Q), B–(P), C–(R), D–(S)

(2) A–(R), B–(Q), C–(S), D–(P)

(3) A–(Q), B–(R), C–(P), D–(S)

(4) A–(R), B–(Q), C–(P), D–(S)

Answer (3)

Sol. $H_2O_2 \rightarrow H_2O + O_2$: Disproportionation reaction

$NaH \rightarrow Na + H_2$: Decomposition reaction

$CH_4 + O_2 \rightarrow CO_2 + H_2O$: Combustion reaction

$Fe + CuSO_4 \rightarrow FeSO_4 + Cu$: Displacement reaction

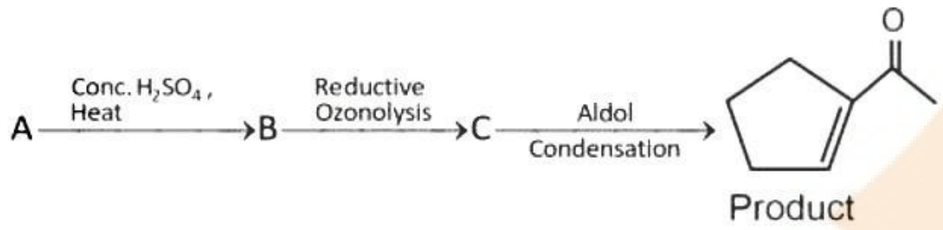
8. Among the following the incorrect order of atomic radius is

(1) B > Al > Mg > F (2) Al > B > N > F

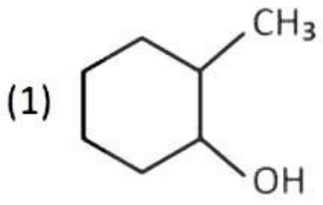
(3) Mg > Al > Be > O (4) Mg > Be > N > F

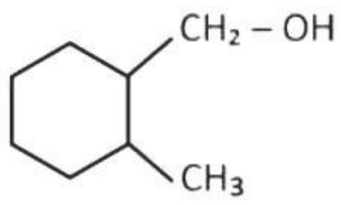
Answer (1)

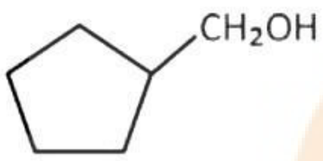
Sol. As we move down the group size increases whereas on moving left to right in a period size decreases

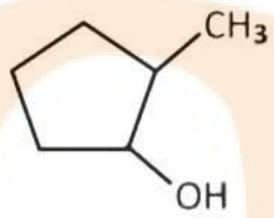
9. 

Identify the Compound A

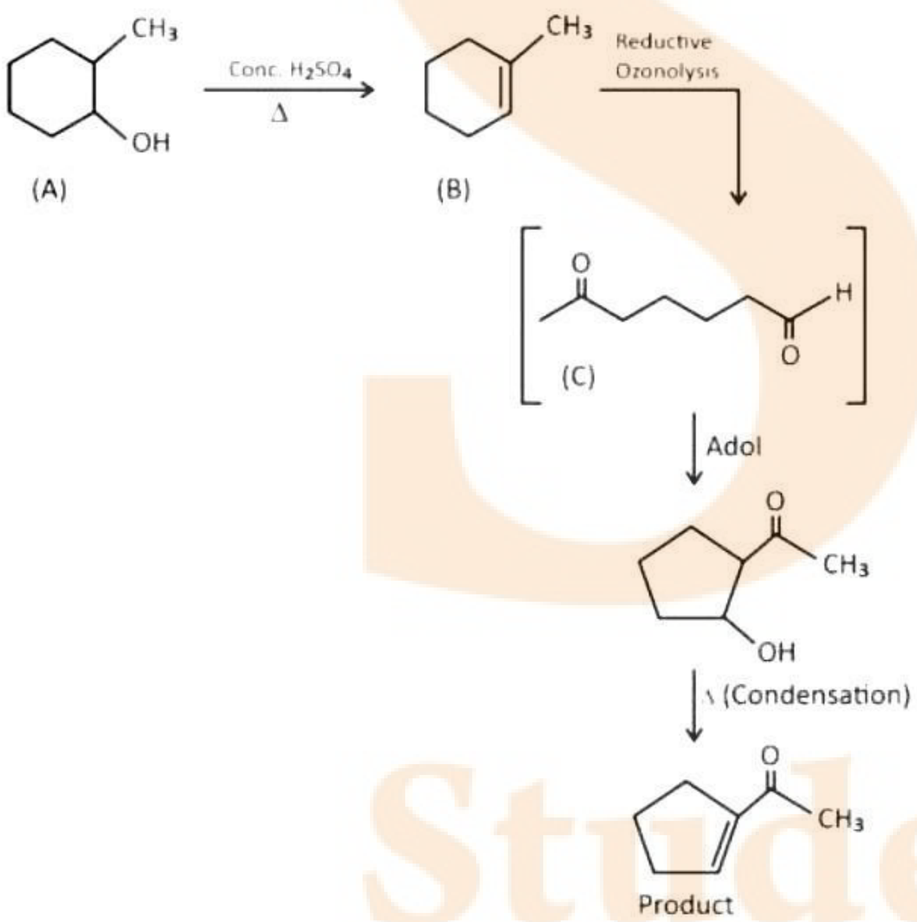
(1) 

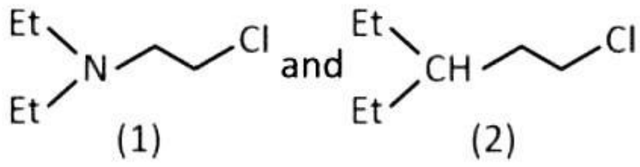
(2) 

(3) 

(4) 

Answer (1)

Sol. 

10. 

Statement-I: Compound (2) shows faster alkaline hydrolysis compared to (1).

Statement-II: Compound (1) shows substitution via neighbouring group participation.

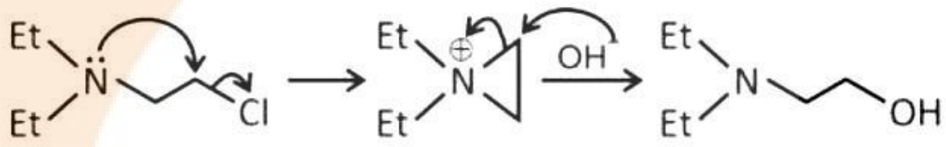
(1) Statement-I is correct and statement-II is incorrect

(2) Statement-I is incorrect and statement-II is correct

(3) Statement-I and statement-II both are correct

(4) Statement-I and statement-II both are incorrect

Answer (2)

Sol. 

11. Which of the following has same energy in absence of electric and magnetic field for hydrogen atom?

(1) 2s, 3p (2) 3s, 2p

(3) 2s, 2p (4) 3s, 4f

Answer (3)

Sol. For hydrogen atom in absence of electric and magnetic field, energy only depends on principal quantum number(n). For same the value of n, energy will be same

Hence 2s and 2p have same energy.

12. Which of the following reaction(s)/test(s) can be used to distinguish acetaldehyde and acetone?

(A) Iodoform Test (B) Cannizzaro

(C) Aldol Condensation (D) Fehling's Test

(E) Tollen's Test (F) Clemmensen's Reduction

(1) (D), (E) Only

(2) (A), (B), (C), (F) only

(3) (B), (C), (F) only

(4) (B), (C), (D), (E) only

Answer (1)

Sol. CH_3CHO will react with Fehling's solution and Tollen's reagent while CH_3COCH_3 will not. Iodoform, Aldol and Clemmensen's reduction will be shown by both

13. Which of the following give violet colour in Borax bead test?

(1) Cr^{3+}

(2) Mn^{2+}

(3) Co^{3+}

(4) Fe^{2+}

Answer (2)

Sol. Colour of Borax bead in oxidising flame is violet for Mn^{2+}.

14. Which of the following compounds have the same number of lone pair on central atom as ClF_3.

(1) XeF_5^-

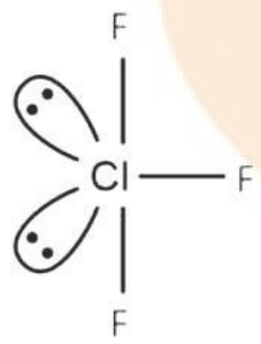
(2) XeF_2

(3) BrF_5

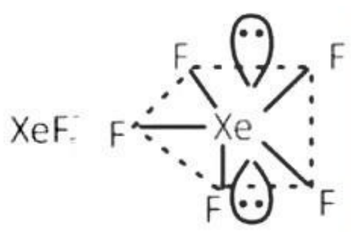
(4) I_3^-

Answer (1)

Sol. $ClF_3 \Rightarrow$



No. of lone pairs on central atom = 2



No. of lone pairs on central atom = 2

$XeF_2 \Rightarrow$ no. of lone pairs on central atom = 3

$I_3^- \Rightarrow$ no. of lone pairs on central atom = 3

$BrF_5 \Rightarrow$ no. of lone pair on central atom-1

15. **Statement 1:** For titration of oxalic acid using $KMnO_4$, warming of acid solution is required whereas in case of Ferrous Ammonium Sulphate, it is done at room temperature.

Statement 2: Fe^{2+} converts to Fe^{3+} during titration.

(1) Both Statement 1 and statement 2 are correct

(2) Statement 1 is correct and statement 2 is incorrect

(3) Statement 1 is incorrect and statement 2 is correct

(4) Both Statement 1 and statement 2 both are incorrect

Answer (1)

Sol. In case of oxalic acid warming of solution (50°C-60°C) is done as rate of reaction is very slow at room temperature.

$$Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Δ_fH of H(g) is 222 kJ/mol, Δ_fH of O(g) is 250 kJ/mol, Δ_fH of $H_2O(g)$ is –248 kJ/mol. What is the value of Bond Energy of O – H bond in H_2O in kJ/mol?

Answer (471)

Sol. $$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g)$$

$$\Delta H_f = \sum(\text{B.E. of reactant}) - \sum(\text{B.E. of product})$$

$$-248 = 444 + 250 - (2 \times B.E._{O-H})$$

$$2B.E._{O-H} = 942$$

$$2B.E._{O-H} = 471\,kJ/mole$$

22. 70% by mass solution of HNO_3 is taken having density 1.41 g/mL. Calculate the molarity of solution

Answer (16)

Sol. $$M = \frac{10xd}{M_0}$$

$$= \frac{10 \times 70 \times 1.41}{63}$$

= 15.66 M

23. 1g of a non-electrolyte solute (MW = 256 g/mol) is dissolved in 50g of solvent, freezing point of solution is lowered by 0.40 K. Calculate the Molal depression constant of solvent.

Answer (5)

Sol. $\Delta T_f = ik_f \times m$

$$\Delta T_f = ixk_f \times \frac{W_{solute}}{MW_{solute} \times W_{solvent}} \times 1000$$

$$0.4 = k_f \times \frac{1 \times 1000}{256 \times 50}$$

$$k_f = \frac{0.4 \times 256 \times 50}{1000}$$

k_f = 5.12 K kg mol^{-1}

24. A compound contains 14.4% carbon, 1.2% hydrogen and 84.4% chlorine, calculate empirical formula mass of compound.

(Molar mass of C = 12, H = 1, Cl = 35.5]

Answer (84)

Sol. Let mass of compound = 100g

	Mole	Molar ratio
C = 14.4g $\Rightarrow$	$\frac{14.4}{12} = 1.2$	1
H = 1.2g $\Rightarrow$	$\frac{1.2}{1} = 1.2$	1
Cl = 84.4g $\Rightarrow$	$\frac{84.4}{35.5} = 2.4$	2

E.F. = $CHCl_2$

E.F. mass = 84

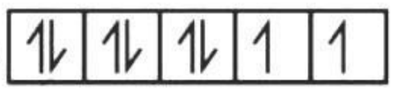
25. How many of the following ions have same value of spin only magnetic moment?

Ni^{2+}, V^{2+}, Ti^{2+}, Sc^{3+}, Ti^{3+}

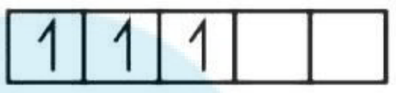
Answer (2)

Sol. The ions having same number of unpaired electrons having same value of spin only magnetic moment.

$\mu = \sqrt{n(n+2)}$ BM (n = Number of unpaired electron)

$Ni^{2+} \Rightarrow 3d^8 \Rightarrow$ 

n = 2

$V^{2+} \Rightarrow 3d^3 \Rightarrow$ 

n = 3

$Ti^{2+} \Rightarrow 3d^2 \Rightarrow$ 

n = 2

$Sc^{3+} \Rightarrow 3d^0$

n = 0

$Ti^{3+} \Rightarrow 3d^1 \Rightarrow$ 

n = 1

Ni^{2+} and Ti^{2+} have same value of spin only magnetic moment.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{1+e^x}dx = \pi(\alpha\pi^2+\beta)$, then $(\alpha+\beta)^2$ is

(1) 100 (2) 144

(3) 169 (4) 196

Answer (1)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{1+e^x}dx$

$= \int_0^{\frac{\pi}{2}} 96x^2\cos^2 x\,dx$

$= 48\int_0^{\frac{\pi}{2}} x^2(1+\cos 2x)dx$

$= 48\left(\frac{x^3}{3}\right)\Big|_0^{\frac{\pi}{2}} + 48\int_0^{\frac{\pi}{2}} x^2\cos 2x dx$

$= 16\left[\frac{\pi^2}{8}\right] + 48\left[\frac{x^2\sin 2x}{2}\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(2x)\left(\frac{\sin 2x}{2}\right)dx\right]$

$= 2\pi^2 + 48(0-0) - 48\int_0^{\frac{\pi}{2}} x\sin 2x\,dx$

$= 2\pi^3 + 48\left[\frac{-x\cos x}{2}\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(1)\left(\frac{-\cos 2x}{2}\right)dx\right]$

$= 2\pi^3 - 48\left[\left(\frac{\pi}{2}\right)\times\frac{1}{2}\right] + 48\int_0^{\frac{\pi}{2}} \frac{\cos 2x}{2}dx$

$= 2\pi^2 - 12\pi + 24\left(\frac{\sin 2x}{2}\right)\Big|_0^{\frac{\pi}{2}}$

$= 2\pi^2 - 12\pi + (0-0) = \pi\left[2\pi^2 - 12\right]$

$= \pi\left[\alpha\pi^2+\beta\right]$

$\Rightarrow \alpha = 2, \beta = -12$

$\Rightarrow (\alpha+\beta)^2 = (2-12)^2 = 100$

2. Number of ways to form 5 digit numbers greater than 50000 with the use of digits 0, 1, 2, 3, 4, 5, 6, 7 such that sum of first and last digit is not more than 8, is equal to

(1) 5119 (2) 5120

(3) 4607 (4) 4608

Answer (3)

Sol. a b c d e $a \geq 5$

$a + e \leq 8$

not all b, c, d, e are zero

if $a = 5$.

$\Rightarrow$ (i) $a = 5$

$5\_\_\_e$ $\Rightarrow e \in \{3, 2, 1, 0\}$

$8^3\cdot 4 - 1$ ways

$\Rightarrow$ (ii) $a = 6$

$6\_\_\_e$ $\Rightarrow e \in \{2, 1, 0\}$

$8^3\cdot 3$ ways

$\Rightarrow$ (iii) $a = 7$

$7\_\_\_e$ $\Rightarrow e \in \{1, 0\}$

$\Rightarrow 8^3\cdot 2$ ways

$\Rightarrow$ Total ways

$\Rightarrow 8^3(2+3+4) - 1 = 4608 - 1 = 4607$

3. If the image of the point $P(4, 4, 3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{1}$ is $Q(\alpha, \beta, \gamma)$. Then $(\alpha + \beta + \gamma)$ is equal to

(1) 7

(2) $\frac{31}{3}$

(3) $\frac{11}{3}$

(4) 8

Answer (2)

Sol. $P(4, 4, 3)$

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{1}=\lambda$$

Any pint of line $R(2\lambda + 1, \lambda + 2, \lambda + 1)$

$$\overrightarrow{PR}:(2\lambda-3)\hat{i}+(\lambda-2)\hat{j}+(\lambda-2)\hat{k}$$

$$\overrightarrow{PR}<2, 1, 1>=0$$

$2(2\lambda - 3) + (\lambda - 2) + 2(\lambda - 2) = 0$

$6\lambda = 10$

$$\lambda=\frac{5}{3}$$

$$\therefore\ R\left(\frac{13}{3}, \frac{11}{3}, \frac{8}{3}\right)$$

Now, $Q(\alpha, \beta, \gamma)$

$$\frac{\alpha+4}{2}=\frac{13}{3}, \frac{\beta+4}{2}=\frac{11}{3}, \frac{\gamma+3}{2}=\frac{8}{3}$$

$$\alpha=\frac{14}{3},\ \beta=\frac{10}{3}, \gamma=\frac{7}{3}$$

$$\alpha+\beta+\gamma=\frac{14+10+7}{3}=\frac{31}{3}$$

4. If $\int_0^x tf(t)dt = x^2 f(x)$ and $f(2) = 3$, then $f(6)$ equals to

(1) 1 (2) 6

(3) 3 (4) 2

Answer (1)

Sol. $\int_0^x tf(t)dt = x^2 f(x)$

Differentiating both sides w.r.t 'x'

$xf(x) = x^2f'(x) + 2xf(x)$

$$\frac{x^2dy}{dx}+xy=0$$

$$\frac{dy}{y}=\frac{-dx}{x}$$

$\ln y + \ln x = \ln c$

$yx = c$

As $f(2) = 3$

$6 = c$

$\therefore\ yx = 6$

$\therefore$ Put $x = 6$

$y(6) = 6$

$y = 1$

Option (1) is correct

5. Let R be a relation such that $R = \{(x, y) : x, y \in Z$ and $(x + y)$ is even$\}$, then the relation R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive and transitive but not symmetric

(3) Transitive only

(4) Equivalence relation

Answer (4)

Sol. for reflexive

If $(x, x) \in Z$

R : $x + x + 2x$ $\Rightarrow$ R is relexive

For symmetric

If $(x, y) \in R$ $\Rightarrow$ $x + y$ = even

$\Rightarrow$ $y + x$ = even $6(y, x) \in R$

$\Rightarrow$ R is symmetric

If $(x, y) \in R$ $\Rightarrow$ $x + y$ = even

$(y, z) \in R$ $\Rightarrow$ $y + z$ = even

$\Rightarrow$ $x + 2y + Z \in$ even $\Rightarrow$ $x + Z =$ even $- 2y$ even

$\Rightarrow$ $\underline{x + z \in \text{even}}$

$\Rightarrow$ $(x, z) \in R$

$\Rightarrow$ R is equivalence relation.

6. Evaluate

$$\cos\left(\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{33}{65}\right)\right)$$

(1) 0

(2) 1

(3) $\cos\frac{5}{13}$

(4) 2

Answer (1)

Sol. $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{33}{65}\right)$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\left[\frac{5}{13}\sqrt{\frac{1-33^2}{65^2}}+\frac{33}{65}\sqrt{1-\frac{5^2}{13^2}}\right]\right)$$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{5}\right)$$

$$=\cos\left(\frac{\pi}{2}\right)=0$$

7. The sum of squares of real roots of the equation: $x^2 + |2x - 3| - 4 = 0$, is

(1) $6\left(2-\sqrt{2}\right)$ (2) $3\left(2-\sqrt{2}\right)$

(3) $3\left(2+\sqrt{2}\right)$ (4) $6\left(2+\sqrt{2}\right)$

Answer (1)

Sol. $x^2 + |2x - 3| - 4 = 0$

(i) $2x - 3 \geq 0$ $\Rightarrow$ $x \geq \frac{3}{2}$

$\Rightarrow$ $x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$ $\Rightarrow$ $(x + 1)^2 = 8$

$\Rightarrow$ $x = \pm 2\sqrt{2} - 1$

$\Rightarrow$ $x = \left(2\sqrt{2} - 1\right)$

as $-2\sqrt{2} - 1 < \frac{3}{2}$

(ii) $x \leq \frac{3}{2}$ $\Rightarrow$ $x^2 - (2x - 3) - 4 = 0$

$\Rightarrow$ $x^2 - 2x - 1 - 0 = 0$ $\Rightarrow$ $(x - 1)^2 = 2$

$\Rightarrow$ $x = \pm 2\sqrt{2} + 1$ $\Rightarrow$ $x = -2\sqrt{2} + 1$

as $\sqrt{2} + 1 > \frac{3}{2}$

$\Rightarrow$ two roots are $x = -\sqrt{2} + 1, 2\sqrt{2} - 1$

$\Rightarrow$ Sum of squares = $12 - 6\sqrt{2} = 6\left(2 - \sqrt{2}\right)$

8. Area enclosed by

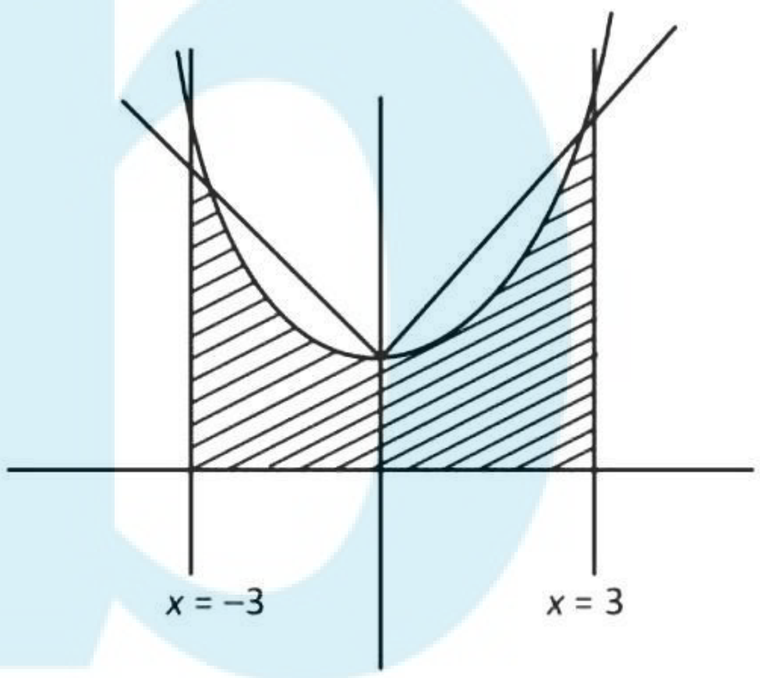
$\left\{(x, y) : 0 \leq y \leq 2|x| + 1, 0 \leq y \leq x^2 + 1, |x| \leq 3\right\}$ is equals to

(1) $\frac{17}{3}$ (2) $\frac{32}{3}$

(3) $\frac{64}{3}$ (4) $\frac{80}{3}$

Answer (3)

Sol.



$$\text{Area} = 2\left[\int_0^2 (x^2 + 1)dn + \frac{1}{2}[5 + 7] \times 1\right]$$

$$= \frac{64}{3}$$

9. There are 2 bad oranges mixed with 7 good oranges and 2 oranges are drawn at random. Let X be the number of bad oranges. The variance of X is

(1) $\frac{51}{268}$ (2) $\frac{49}{162}$

(3) $\frac{63}{108}$ (4) $\frac{91}{206}$

Answer (2)

Sol.

X	0	1	2
$P(X)$	$\frac{^7C_2}{^9C_2}$	$\frac{^7C_1 \cdot ^2C_1}{^9C_2}$	$\frac{^2C_2}{^9C_2}$

$$\text{Variance} = 0^2 \cdot \frac{^7C_2}{^9C_2} + 1^2 \cdot \frac{^7C_1 \cdot ^2C_1}{^9C_2} + 2^2 \cdot \frac{^2C_2}{^9C_2}$$

$$-\left(\frac{0 \cdot ^7C_2}{^9C_2} + \frac{1 \cdot ^7C_1 \cdot ^2C_1}{^9C_2} + \frac{2 \cdot ^2C_2}{^9C_2}\right)^2$$

$$= \frac{7}{18} + \frac{4}{36} - \left(\frac{7}{18} + \frac{2}{36}\right)^2$$

$$= \frac{49}{162}$$

10. Let $f(x) = \frac{2^x}{2^x + \sqrt{2}}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to

(1) $\frac{81}{2}$ (2) 41

(3) $41\sqrt{2}$ (4) 81

Answer (1)

Sol. $f(x) = \frac{2^x}{2^x + 2^{1/2}} = \frac{2^x}{2^x + \sqrt{2}}$

$$f(1-x) = \frac{2^{1-x}}{2^{1-x} + 2^{1/2}} = \frac{\frac{2}{2^x}}{\frac{2}{2^x} + 2^{1/2}} = \frac{2}{2 + \sqrt{2}\,2^x}$$

$$= \frac{\sqrt{2}}{2^x + \sqrt{2}}$$

$$\Rightarrow f(x) + f(1-x) = \frac{\sqrt{2} + 2^x}{\sqrt{2} + 2^x} = 1$$

$$\Rightarrow \sum_{k=1}^{81} f\left(\frac{k}{82}\right) + f\left(\frac{2}{82}\right) + \left(f\left(\frac{3}{82}\right)\right) + \ldots.$$

$$\ldots + f\left(\frac{40}{82}\right) + f\left(\frac{41}{82}\right) + f\left(\frac{42}{82}\right)$$

$$+ \ldots + f\left(\frac{79}{82}\right) + f\left(\frac{80}{82}\right) + f\left(\frac{81}{82}\right)$$

$$= \left[f\left(\frac{1}{82}\right) + f\left(\frac{81}{82}\right)\right] + \left[f\left(\frac{2}{82}\right) + f\left(\frac{80}{82}\right)\right] + \ldots$$

$$+ \left[f\left(\frac{40}{82}\right) + f\left(\frac{42}{82}\right) + f\left(\frac{41}{82}\right)\right]$$

$$= \underbrace{1 + 1 + \ldots. + 1}_{40 \text{ times}} + f\left(\frac{1}{2}\right)$$

$$= 40 + \frac{\sqrt{2}}{\sqrt{2} + \sqrt{2}} = 40 + \frac{1}{2} = \frac{81}{2}$$

11. If $2a_{n+2} = 5a_{n+1} - 3a_n$, where $n = 0, 1, 2, \ldots$. If $a_0 = 3$ and $a_1 = 4$, then the value of $\sum_{k=1}^{100} a_k$ is equal to

(1) $3a_{100} - 91$ (2) $3a_{99} - 91$

(3) $3a_{100} + 91$ (4) $3a_{99} + 91$

Answer (1)

Sol. $2a_{n+2} = 5a_{n+1} - 3a_n = 0$

$$\Rightarrow 2t^2 - 5t + 3 = 0$$

$$\Rightarrow t = 1, \frac{3}{2}$$

$$\therefore \quad a_n = A \cdot (1)^n + B \cdot \left(\frac{3}{2}\right)^2$$

$$a_0 = 3,\ a_1 = 4$$

$$\therefore \quad A = 1,\ B = 2$$

$$a_n = 1 + 2 \cdot \left(\frac{3}{2}\right)^n$$

$$S_{100} = 100 - 6\left(1 - \left(\frac{3}{2}\right)^{100}\right)$$

$$= 3a_{100} - 99$$

Using $\Rightarrow \left(\frac{3}{2}\right)^{100} = \left(\frac{a_{100}-1}{2}\right)$

$a_{100} = 2\cdot\left(\frac{3}{2}\right)^{100} + 1$

12. Let k_1 and k_2 be two randomly selected natural numbers.

The probability that $(i)^{k_1} + (i)^{k_2}$ is non-zero is (where $i = \sqrt{-1}$)

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{1}{4}$ (4) $\frac{1}{6}$

Answer (2)

Sol. $(i)^{k_1} + (i)^{k_2}$ is non zero

$\Rightarrow$ $k_1 : 4\lambda_1 + r_1, \quad r_1 \in \{0, 1, 2, 3\}$

$K_2 : 4\lambda_2 + r_2, \quad r_2 \in \{0, 1, 2, 3\}$

The pairs to get zero will be

$(1, -1), (i, -i)$

$\Rightarrow$ (i) $(1, -1)$ pair

$\Rightarrow (r_1, r_2) \in \{(2, 0), (0, 2)\}$

(ii) $(i, -i)$ pair

$\Rightarrow (r_1, r_2) \in \{(1, 3), (3, 1)\}$

$\Rightarrow$ probablity $\left(i^{k_1} + i^{k_2} \neq 0\right)$

= 1 – probablity $\left(i^{k_1} + i^{k_2} \neq 0\right)$

$= 1 - \frac{4}{16} = \frac{12}{16} = \frac{3}{4}$

13. In ΔABC, $A(4\sin\theta, 4\cos\theta)$, $B(-2\cos\theta, 0)$ and $C(2, 2\sin\theta)$. If locus of centroid is $(3x - 2)^2 + (3y)^2 = \alpha$, then α is

(1) 20

(2) 4

(3) 16

(4) 12

Answer (1)

Sol. $h = \frac{4\sin\theta - 2\cos\theta + 2}{3} \Rightarrow 3h - 2 = 4\sin\theta - 2\cos\theta$ ...(1)

$k = \frac{4\cos\theta - 2\sin\theta}{3} \Rightarrow 3k = 4\cos\theta + 2\sin\theta$ ...(2)

$(1)^2 + (2)^2$

$(3h-2)^2 + (3k)^2 = (4\sin\theta - 2\cos\theta)^2 + (4\cos\theta + 2\sin\theta)^2$

$= 16\sin^2\theta + 4\cos^2\theta - 8\sin\theta\cos\theta +$

$16\cos^2\theta + 4\sin^2\theta - 8\sin\theta\cos\theta$

$(3h - 2)^2 + (3k)^2 = 20$

$(3x - 2)^2 + (3y)^2 = 20$

14. Let $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse and a series of ellipse are drawn that E_{i+1} has same centre, eccentricity as E_i and E_{i+1}'s major axis is minor axis of E_i. If S_i be the area of E_i, then $\left(\frac{5}{\pi}\sum_{i=1}^{\infty} S_i\right)$ is equal to

(1) 63 (2) 54

(3) 45 (4) 72

Answer (2)

Sol. Let b_i be minor axis of E_i

a_i be major axis of E_i

$\Rightarrow e_i = 1 - \frac{b_i^2}{a_i^2}$

Now, b_{i+1} minor axis of E_{i+1}

A_{i+1} major axis of E_{i+1}

$\Rightarrow e_{i+1} = 1 - \frac{b_{i+1}^2}{a_{i+1}^2}$, also $a_{i+1} = b_i$ and $e_i = e_{i+1}$

$= \frac{b_i^2}{a_i^2} = \frac{b_{i+1}^2}{(b_i)^2} \Rightarrow b_{i+1} = \frac{b_i^2}{a_i}$

$\Rightarrow$ Area of $E_i = S_i = \pi a_i b_i$

$\Rightarrow S_{i+1} = \pi a_{i+1} b_{i+1}$

$= \pi(b_i)\left(\frac{b_i^2}{a_i}\right)$

$$= \pi(b_i a_i)\underbrace{\left(\frac{b_i}{a_i}\right)^2}$$

$$S_{i+1} = S_i(1-e_i^2)$$

$$\Rightarrow \quad S_{i+1} = S_i\left(1-\left(1-\frac{4}{9}\right)\right) = S_i \cdot \frac{4}{9}$$

$$\Rightarrow \quad S_i = 6\pi, S_2 = 6\pi \cdot \frac{4}{9} S_3 = 6\pi \cdot \left(\frac{4}{9}\right)^2$$

$$\Rightarrow \quad \sum_{k=1}^{\infty} S_i = \left(\frac{6\pi}{1-\frac{4}{9}}\right) = \frac{54\pi}{5}$$

$$\Rightarrow \quad \frac{5}{\pi}\sum_{k=1}^{\infty} S_i = \frac{5}{\pi} \cdot \frac{54\pi}{5} = 54$$

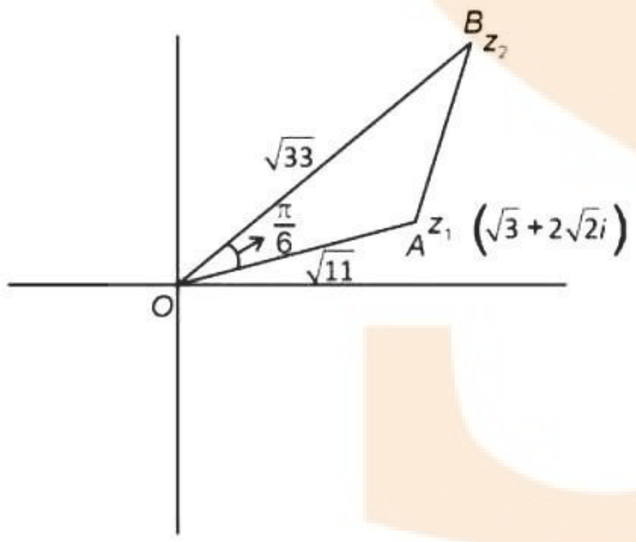
15. Let $z_1 = \sqrt{3} + 2\sqrt{2}i$ and $\sqrt{3}\,|z_1| = |z_2|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$, then the area of triangle with vertices z_1, z_2 and origin is (in sq. units)

(1) $\frac{11\sqrt{3}}{4}$ (2) $\frac{11\sqrt{2}}{3}$

(3) $\frac{11}{4}$ (4) $\frac{2\sqrt{2}}{3}$

Answer (1)

Sol.



Area of $\Delta OAB = \frac{1}{2} \times \sqrt{11} \times \sqrt{33} \sin\frac{\pi}{6}$

$= \frac{11\sqrt{3}}{4}$ square units

16. Let $f(x) = \begin{cases} 2x, & x < 0 \\ \min(1+x+[x], 1+2[x]), & 0 \le x < 2 \\ 5, & x \ge 2 \end{cases}$

If α is the number of points of discontinuity and β is the number of points of non-differentiability, then $(\alpha + \beta)$ is equal to (where [.] denote greatest integer function)

(1) 6 (2) 5

(3) 4 (4) 8

Answer (1)

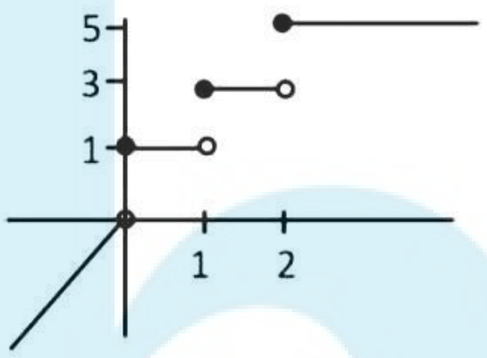
Sol. $f(x) = \begin{cases} 2x, & x < 0 \\ \min(1+x+[x], 1+2[x]), & 0 \le x < 2 \\ 5 & x \ge 2 \end{cases}$

$1 + x + [x] = 1 + \{x\} + 2[x]$

Since $\{x\} \ge 0 \ \forall x \in R$

$\Rightarrow \quad 1 + x + [x] \ge 1 + 2[x]$

$\Rightarrow \quad f(x) = \begin{cases} 2x & , x < 0 \\ 1+2[x], & 0 \le x < 0 \\ 5 & x \ge 2 \end{cases}$



Number of discontinuity = 3 $\Rightarrow \alpha = 3$

Number of point of non-differentiability

$= 3 \Rightarrow \beta = 3$

17. If $\alpha = 1 + \sum_{n=1}^{6}(-3)^{n-1} \cdot {}^{12}C_{2n-1}$, then distance of point $\left(12, \sqrt{3}\right)$ from the line $\alpha x - \sqrt{3}y + 100 = 0$ is,

(1) $\frac{109}{2}$

(2) 55

(3) 54

(4) 109

Answer (1)

Sol. $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \cdot {}^{12}C_{2r-1}$

$= 1 + \sum_{r=1}^{6} \left[\left(\sqrt{3}i\right)^2 \right]^{r-1} \cdot {}^{12}C_{2r-1}$

$= 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^{6} \left(\sqrt{3}i\right)^{2r-1} \cdot {}^{12}C_{2r-1}$

Let $\sqrt{3}i = x$

$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^{6} {}^{12}C_{2r-1} \cdot x^{2r-1}$

$\alpha = 1 + \frac{1}{\sqrt{3}i} \left[{}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + {}^{12}C_5 \cdot x^5 + \ldots + {}^{12}C_{11} \cdot x^{11} \right]$

Let $(1+x)^{12} = {}^{12}C_0 \cdot x^0 + {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 + \ldots + {}^{12}C_{12} \cdot x^{12}$

$(1-x)^{12} = {}^{12}C_0 \cdot x^0 - {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 - {}^{12}C_3 \cdot x^3 \ldots + {}^{12}C_{12} \cdot x^{12}$

$(1+x)^{12} - (1-x)^{12} = 2\left({}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + \ldots + {}^{12}C_{11} \cdot x^{11} \right)$

$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i} \left[\frac{(1+x)^{12} - (1-x)^{12}}{2} \right]$

$\alpha = 1 + \frac{1}{\sqrt{3}i} \left[\frac{\left(1+\sqrt{3}i\right)^{12} - \left(1-\sqrt{3}i\right)^{12}}{2} \right]$

Since, $\omega = \frac{-1}{2} + \frac{\sqrt{3}i}{2} \quad \Rightarrow \quad -2\omega = 1 - \sqrt{3}i$

$\omega^2 = \frac{-1}{2} - \frac{\sqrt{3}i}{2} \quad \Rightarrow \quad -2\omega^2 = 1 + \sqrt{3}i$

$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i} \left(\frac{\left(-2\omega^2\right)^{12} - (2\omega)^{12}}{2} \right)$

$= 1 + \frac{2^{11}}{\sqrt{3}i} \left(\omega^{24} - \omega^{12} \right)$

$= 1 + \frac{2^{11}}{\sqrt{3}i} (1-1) = 1$

$\alpha = 1, \Rightarrow$ perpendicular distance from $\left(12, \sqrt{3}\right)$ is

$\frac{\left|12 - \sqrt{3}\left(\sqrt{3}\right) + 100\right|}{\sqrt{1^2 + \left(\sqrt{3}\right)^2}} = \frac{109}{2}$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In an AP, $T_m = \frac{1}{25}, T_{25} = \frac{1}{20}$ and $20\sum_{r=1}^{25} T_r = 13$, then

$5m \sum_{r=m}^{2m} T_r$ equals

Answer (126)

Sol. $T_m = a + (m-1)d \cdot \frac{1}{25} \quad \ldots(1)$

$T_{25} = a + 24d = \frac{1}{20}$

$20 \times \frac{2r}{2} \left[a + \frac{1}{20} \right] = 13 \Rightarrow a = \frac{1}{20 \times 25}$

$20\sum_{r=1}^{25} T = 20 \times \frac{25}{2}[2a + 24d] = 13$

$d = \frac{1}{20 \times 25}$

Substitute a and d in (i)

$\Rightarrow m = 20$

Now $5m \sum_{r=m}^{2m} T_r = 5 \times 20 \left[\sum_{r=20}^{40} T_r \right]$

$$= 100\left[\frac{40}{2}[2a + 39d]\right] - \frac{19}{2}[2a + 18d]$$

$$= 100\left[\frac{40}{2} \times 41d - \frac{19}{2} \cdot 20d\right]$$

$$= 100\left[\frac{40}{2} \times \frac{41}{20 \times 25} - \frac{19.20}{2 \times 20 \times 25}\right]$$

= 126

22.

23.

24.

25.

❑ ❑ ❑